



**MATHEMATICS  
 HIGHER LEVEL  
 PAPER 1**

Wednesday 5 May 2010 (afternoon)

Candidate session number

2 hours

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**INSTRUCTIONS TO CANDIDATES**

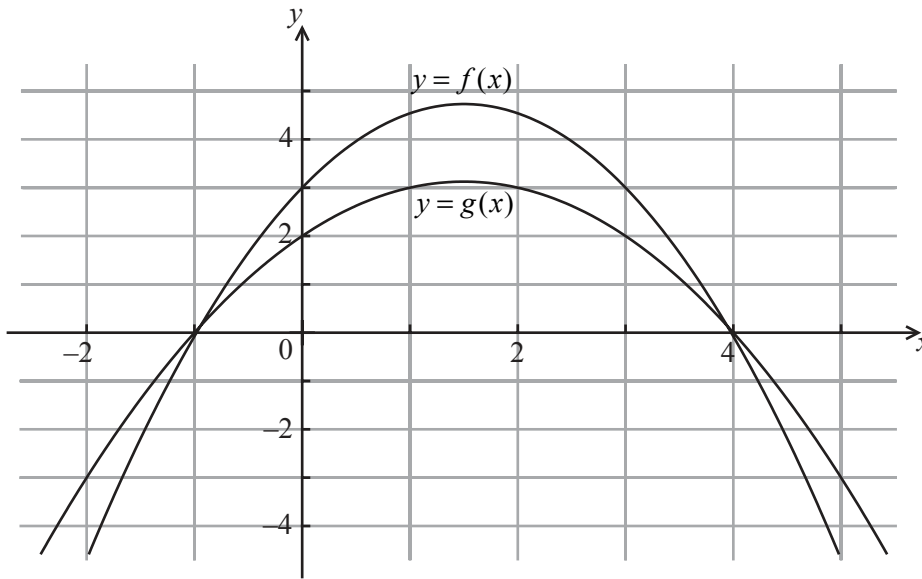
- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.





2. [Maximum mark: 4]

Shown below are the graphs of  $y = f(x)$  and  $y = g(x)$ .



If  $(f \circ g)(x) = 3$ , find all possible values of  $x$ .

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3. [Maximum mark: 7]

(a) Show that the two planes

$$\begin{aligned}\pi_1 : x + 2y - z &= 1 \\ \pi_2 : x + z &= -2\end{aligned}$$

are perpendicular.

[3 marks]

(b) Find the equation of the plane  $\pi_3$  that passes through the origin and is perpendicular to both  $\pi_1$  and  $\pi_2$ .

[4 marks]

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4. [Maximum mark: 5]

Solve the equation  $4^{x-1} = 2^x + 8$ .

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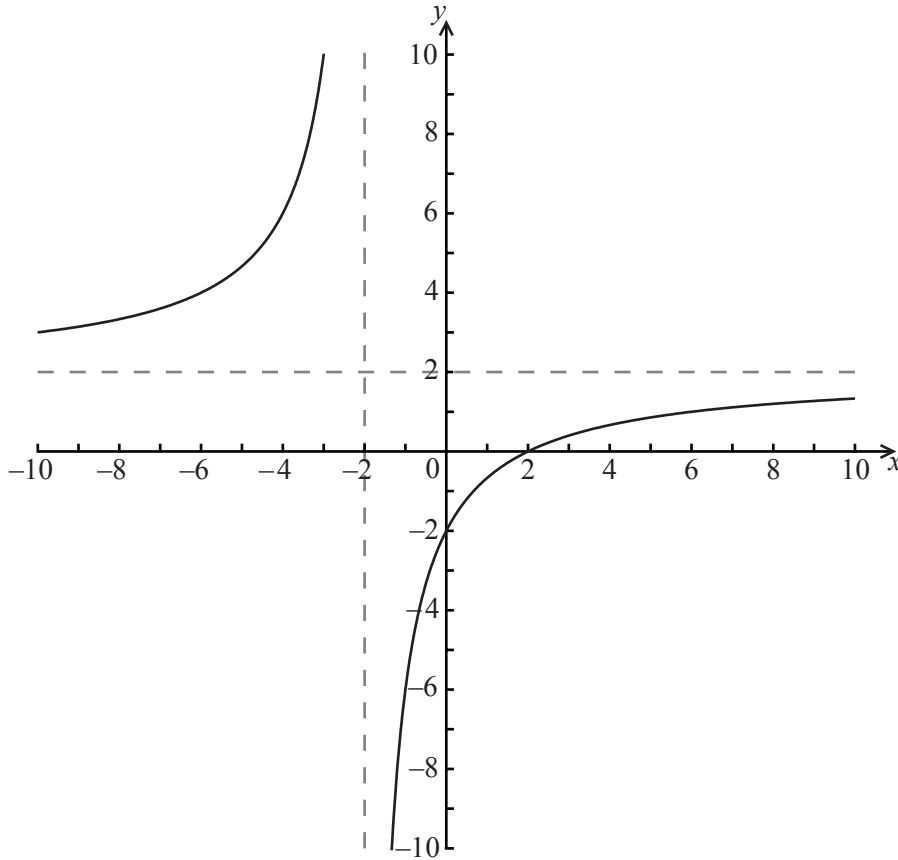
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5. [Maximum mark: 8]

The graph of  $y = \frac{a+x}{b+cx}$  is drawn below.



(a) Find the value of  $a$ , the value of  $b$  and the value of  $c$ .

[4 marks]

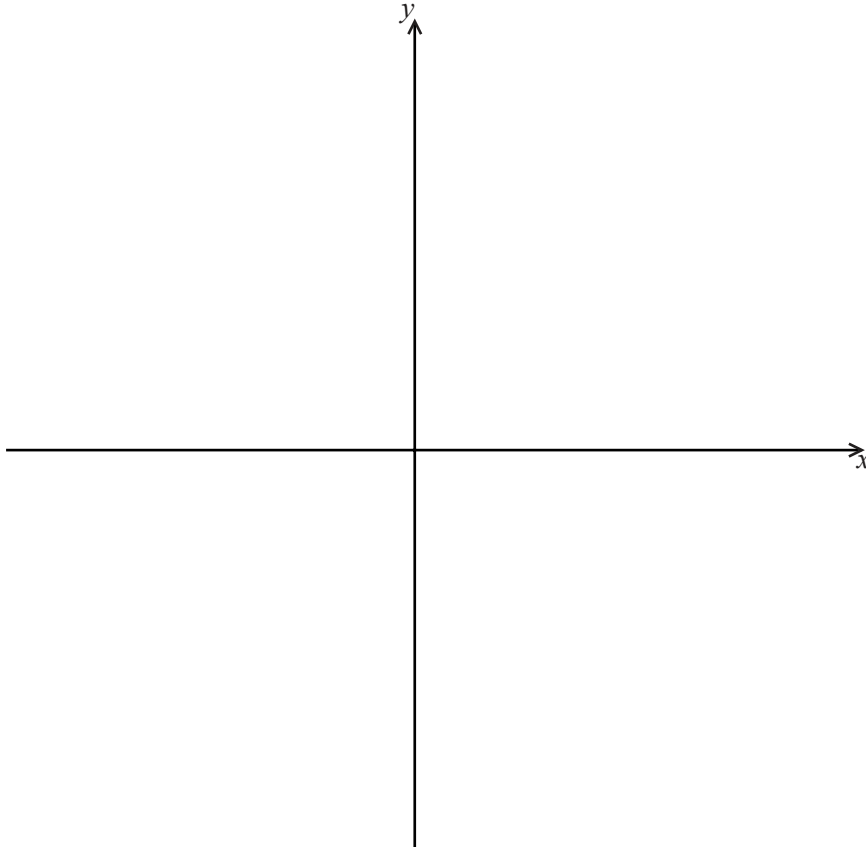
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*(Question 5 continued)*

- (b) Using the values of  $a$ ,  $b$  and  $c$  found in part (a), sketch the graph of  $y = \left| \frac{b+cx}{a+x} \right|$  on the axes below, showing clearly all intercepts and asymptotes. *[4 marks]*



6. [Maximum mark: 4]

Consider the vectors  $\vec{OA} = \mathbf{a}$ ,  $\vec{OB} = \mathbf{b}$  and  $\vec{OC} = \mathbf{a} + \mathbf{b}$ . Show that if  $|\mathbf{a}| = |\mathbf{b}|$  then  $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = 0$ . Comment on what this tells us about the parallelogram OACB.

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7. [Maximum mark: 7]

Two players, A and B, alternately throw a fair six-sided dice, with A starting, until one of them obtains a six. Find the probability that A obtains the first six.

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8. [Maximum mark: 7]

The region enclosed between the curves  $y = \sqrt{x} e^x$  and  $y = e\sqrt{x}$  is rotated through  $2\pi$  about the  $x$ -axis. Find the volume of the solid obtained.

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9. [Maximum mark: 7]

(a) Given that  $\alpha > 1$ , use the substitution  $u = \frac{1}{x}$  to show that

$$\int_1^\alpha \frac{1}{1+x^2} dx = \int_{\frac{1}{\alpha}}^1 \frac{1}{1+u^2} du . \quad [5 \text{ marks}]$$

(b) Hence show that  $\arctan \alpha + \arctan \frac{1}{\alpha} = \frac{\pi}{2}$ . [2 marks]

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10. [Maximum mark: 6]

The ten numbers  $x_1, x_2, \dots, x_{10}$  have a mean of 10 and a standard deviation of 3.

Find the value of  $\sum_{i=1}^{10} (x_i - 12)^2$ .

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**SECTION B**

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

**11.** [Maximum mark: 20]

Consider  $f(x) = \frac{x^2 - 5x + 4}{x^2 + 5x + 4}$ .

- (a) Find the equations of all asymptotes of the graph of  $f$ . [4 marks]
- (b) Find the coordinates of the points where the graph of  $f$  meets the  $x$  and  $y$  axes. [2 marks]
- (c) Find the coordinates of
  - (i) the maximum point and justify your answer;
  - (ii) the minimum point and justify your answer. [10 marks]
- (d) Sketch the graph of  $f$ , clearly showing all the features found above. [3 marks]
- (e) **Hence**, write down the number of points of inflexion of the graph of  $f$ . [1 mark]

**12.** [Maximum mark: 20]

A continuous random variable  $X$  has probability density function

$$f(x) = \begin{cases} 0 & , \quad x < 0 \\ ae^{-ax} & , \quad x \geq 0. \end{cases}$$

It is known that  $P(X < 1) = 1 - \frac{1}{\sqrt{2}}$ .

- (a) Show that  $a = \frac{1}{2} \ln 2$ . [6 marks]
- (b) Find the median of  $X$ . [5 marks]
- (c) Calculate the probability that  $X < 3$  given that  $X > 1$ . [9 marks]



13. [Maximum mark: 20]

(a) Show that  $\sin 2nx = \sin((2n+1)x)\cos x - \cos((2n+1)x)\sin x$ . [2 marks]

(b) Hence prove, by induction, that

$$\cos x + \cos 3x + \cos 5x + \dots + \cos((2n-1)x) = \frac{\sin 2nx}{2 \sin x},$$

for all  $n \in \mathbb{Z}^+$ ,  $\sin x \neq 0$ . [12 marks]

(c) Solve the equation  $\cos x + \cos 3x = \frac{1}{2}$ ,  $0 < x < \pi$ . [6 marks]

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